Chapter 7
SHELLS

7.1. Introduction

The shell is a body formed by two surfaces; the distance *t* between which is small in comparison with the other two dimensions. *t* is the thickness of the shell. The surface that separates equally the thickness is known as midsurface. Geometrically, a shell is described by its thickness and the shape of the shell surface.. The thickness can be constant or changeable. Each point of the midsurface is characterized by two radii of curvature called principal radii of curvature. The centers of curvature lie on a normal to the midsurface in the given point. In general, principal radii vary from point to point

The so called closed shells (Fig. 7.1) are restricted only by two surfaces. If from the closed shell is separate a part with cross section perpendicular to the middle plane, we obtain open shell (Fig. 7.1, b).

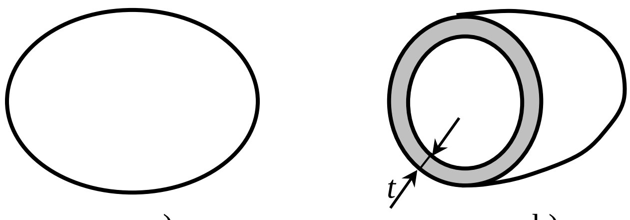


Fig. 7.1

The shell can be called 'curved plate' as well, and its theory is constructed in a way similar to that of the plates. A thin shell is a shell for which

$$R/t \geq 20, \tag{7.1}$$

where *R* is the smallest radius of the curvature of the shell. In addition to this $t << a$, where *a* is a typical dimension of the shell.

Usually the shell undergoes a load in its midsurface and a load perpendicular to it. The shell is more rigid and undergoes the load better, if the loads in the midsurface which produces so called membrane stresses predominate (similar to cable that carries out better open, but not bending). By definition the membrane stresses in a plate are these for which the normal stresses perpendicular to the surface of the plate and any bending stresses are negligibly small.

In the practice, there is no shells which are subjected to only membrane stresses. The basic stresses appear close to the places where loads act, the fixed supports, ribbing and in the places where the changes in the geometry of the shell are rapid.

For the shell theory the same hypothesis as in this for the plates are used. Depending on the conditions of loading and the shape of the shell, two types of theory are used:

a) Non-moment theory of thin shells

This theory is applied in the cases when in the cross section of the shell, the normal (from *N* internal force) and the shearing stresses τ_{xy} are significant (Fig. 7.2).

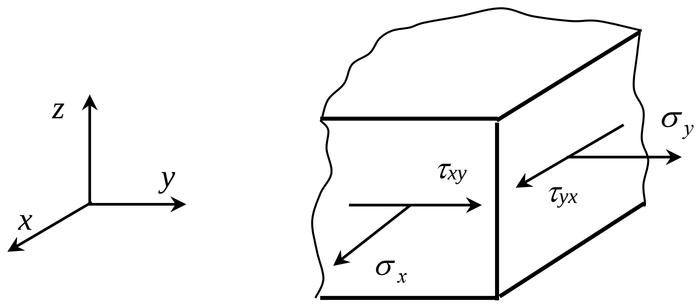


Fig. 7.2

The stress state is similar to that of a plane stress problem. The stresses σ_x, σ_y and τ_{xy} are the so called membrane stresses. Under these constraints the shell does not curve. The relations between the stresses and the internal forces are:

$$N_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x dz \;,\; N_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_y dz \;,\; N_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} dz \,. \tag{7.2}$$

b) Moment theory of shells

Significant are the stresses produced by M_x, M_y, M_{xy}, Q_x and Q_y internal forces, defined as in the plate. The stresses produced by foregoing internal forces, are the so called bending stresses (Fig. 7.3).

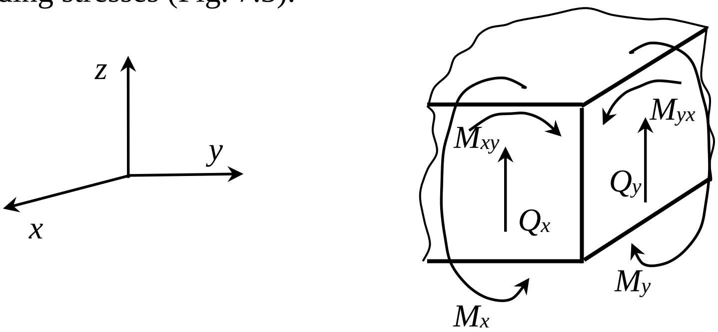
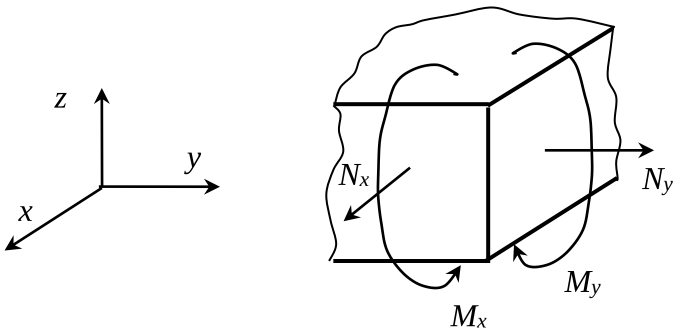


Fig. 7.3

In the general case it is possible to combine the internal forces, and thus the stress using the principle of superposition. For example, in many cases significant are the internal forces M_x, M_y, N_x and N_y and their corresponding stresses (Fig. 7.4).



For the stress σ_x, using the superposition for a thin shell we obtain

$$\sigma_x = \frac{N_x}{t} + \frac{M_x z}{t^3/12} \,. \tag{7.3}$$

Non-moment loading of a shell is the most encountered case, because, due to the small thickness the shell is insensitive to bending. The following conditions are required for non-moment loading:

a) the surface of the shell is smooth and continues, without rapid changes of the curvature

b) the loads are constant (there are no distributed loads)

In most of the cases in the practice we deal with thin shells, for which the classical shell theory is applied. In the case of the thick shells it is necessary to consider the shear deformation as well.

The finite element modeling of the shells is difficult because of the complicated theory. Two methods have been imposed:

a) the combination of a plane element from 2D problems and plate element;

b) the utilization of non-plane elements, constructed on the basis of the classical theory;

7.2. Arched shells. Basic dependences.

It is supposed that the shell is thin and is loaded with lad perpendicular to the middle plane (Fig. 7.5, a). In Fig. 7.5, b is shown an arched element with length *L*. the displacement of a point from the middle line, accordingly on the tangent and perpendicular to it, are shown with *u* and *w*.

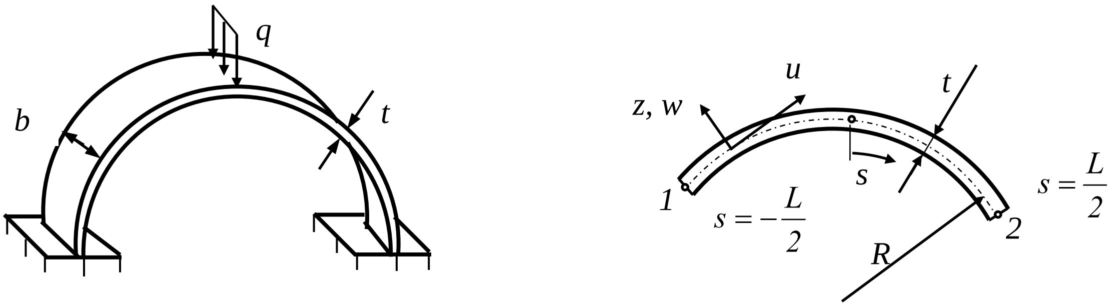


Fig. 7.5

According to the sketches In Fig. 7. 6, for the calculations and the deformations in an arched element with R=*const* can be written the following:

$$\frac{u_1(z)}{R+z}=\frac{u}{R}\ ,\quad \varepsilon_{s,1}=\frac{2\pi(R+z+w)-2\pi(R+z)}{2\pi(R+z)}=\frac{w}{R+z}\approx\frac{w}{R}\ ,\quad u_3(z)=-z\frac{dw}{ds}. \tag{7.4}$$

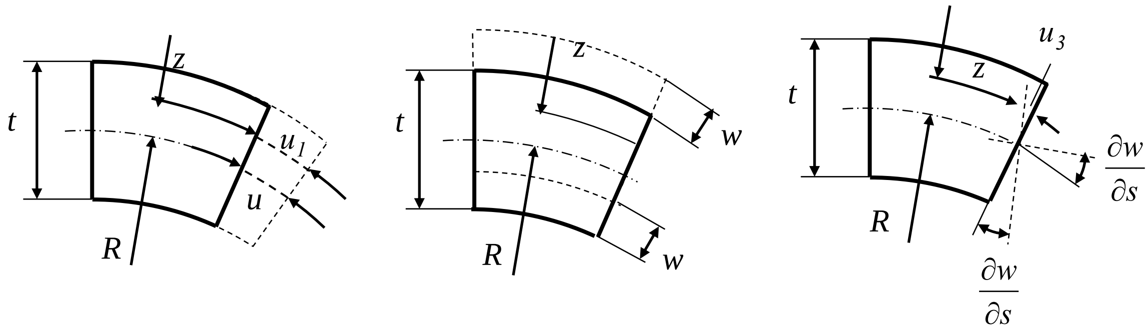


Fig. 7.6

The total displacement of the cross section of the element is obtained by the superposition of the two translational displacements along *s* and *z* (Fig. 7.6, a and 7.6, b) and one rotation (Fig. 7.6, c).

On the basis of (7.4) now it can be written

$$\begin{aligned}\varepsilon_s(z)&=\varepsilon_{s,1}+\frac{d}{ds}(u_1+u_3)=\frac{w}{R}+\frac{d}{ds}(u_1+u_3)=\\&=\frac{du}{ds}+\frac{w}{R}+z\left(\frac{1}{R}\frac{du}{ds}-\frac{d^2w}{ds^2}\right)=\varepsilon_{s,M}+zk_s\end{aligned}\ , \tag{7.5}$$

where

$\varepsilon_M=\frac{du}{ds}+\frac{w}{R}$ is the deformation connected to the membrane forces. The curvature $k=\frac{1}{R}\frac{du}{ds}-\frac{d^2w}{ds^2}$, which is positive in the decrease of the radius of curvature and the deformation connected to it, are a result of the bending. The angular deformation γ_{zs} obtained through the shear of the thin shells is accepted as ignorable small.

The energy of the deformations can be obtained from the expression

$$U = U_{м} + U_{ог} = \int_{-L/2}^{L/2} \frac{1}{2}\sigma_{м}\varepsilon_{м} ds + \int_{V} \frac{1}{2}\sigma_{ог}\varepsilon_{ог} dV = \\ = \int_{-L/2}^{L/2} \frac{EA}{2}\varepsilon_{м}^{2} ds + \int_{-L/2}^{L/2} \frac{EJ}{2}k^{2} ds \quad , \qquad (7.6),$$

where *A* is a cross section area, and $J=bt^3/12$ is axial inertial moment of rectangular cross section with width *b*.

7.2.1. **Modeling of shells with arched element**

The element is shown in Fig. 7.7, a and also is an analogous to plane-beam element.

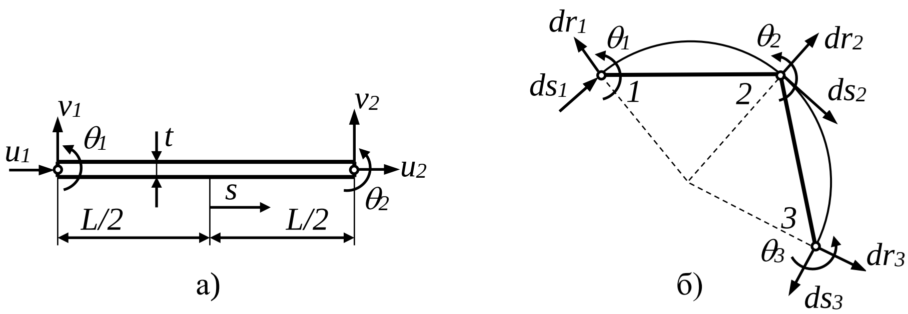


Fig. 7.7

It is obtained through the combination of trussed and beamed elements as

$$\begin{bmatrix} \underset{2x2}{[k_{пр}]} & \underset{2x4}{[0]} \\ \underset{4x2}{[0]} & \underset{4x4}{[k_{гр}]} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \{r\}. \qquad (7.7)$$

For the assembling of the elements can be used, as degrees of freedom, the radial and the tangential displacements $d_{r,i}$ and $d_{s,i}$ in each node (Fig. 7.7, b). The standard transformation is used for the replacement of the nodal values of $d_{r,i}$ and $d_{s,i}$.

7.2.2 Approximation of the displacements

The field of displacement can be approximate with polynomials

$$u = a_1 + a_2 s \\ v = a_3 + a_4 s + a_5 s^2 + a_6 s^3. \qquad (7.8)$$

7.3. Modeling of shells with plane elements

The modeling is carried out with the splitting into the shell to small flat element. When the sizes of the element decrease, a similarity of the solution to correct one is observed. Some examples are shown in Fig. 7.8.

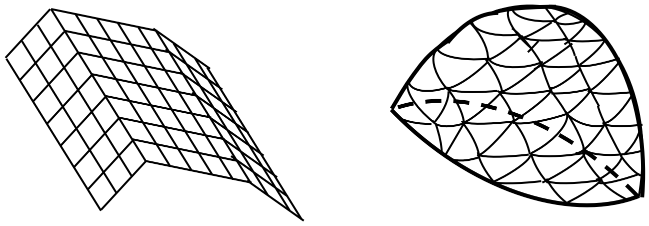


Fig. 7.8

In the general case the elements are under the action of bending and membrane forces. In the plane elements this forces cause independent deformations on condition that the deformations are insignificant. For the construction of shell with an arbitrary form, the most suitable for usage are the triangular elements. In some shells (for example cylindrical) can be rectangular elements as well. In many practical cases the plane elements give good similarity and in addition to that they allow simple connection with supporting elements.

An element working under the action of membrane and bending forces simultaneously can be obtained from the combination of a plane 2D element and a slab one (Fig. 7.9). In this way a shell plane element is obtained.

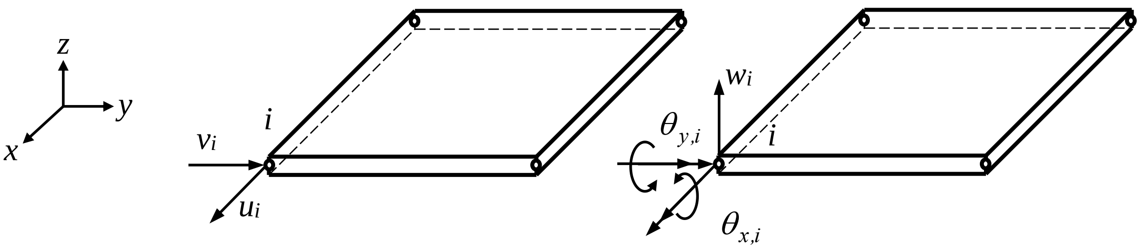


Fig. 7.9

Because it is possible to have elements in different planes, it is necessary to introduce not only a local but a global coordinate system as well. The local coordinate system is chosen in the way that $\bar{x}$ is be on the common line of the elements, and $\overline{xy}$ is in the middle plane of the element. Usually from the global coordinate system y is parallel to $\bar{y}$, and xy is a horizontal plane.

If the construction is spatial, it is necessary along the edges between the elements of different planes to be supplied continuity of the functions of the displacements and the rotations along the three axes, in order to obtain an accurate solution. That's why in each node, that coincides with such edge, is absolutely necessary to be introduced 6 nodal parameters, for example for the node i - $\{\bar{d}_i\}^T = [\bar{u}_i\ \bar{v}_i\ \bar{w}_i\ \theta_{\bar{x},i}\ \theta_{\bar{y},i}\ \theta_{\bar{z},i}]$.

7.3.1. Rectangular 4-noded element with 20 degrees of freedom

The element is shown in Fig. 7.10.

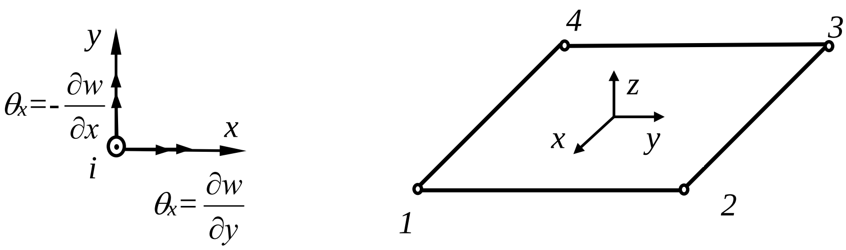


Fig. 7.10

Because the transition to global coordinate system is done with a rectangular transposed matrix, it is easier to make it square (so that the transposed matrix could be obtained). For this reason in each node of the shell element a parameter $\theta_{\bar{z},i}$, is added, around axis perpendicular to the plane of the element. To $\theta_{\bar{z},i}$ should correspond the nodal moment $M_{\bar{z},i}$. So in the local coordinate system are obtained 6-node parameters per node and $\{\bar{d}\}$ is matrix 1x24 and $i = 1,2,3,4$. The vector of the nodal loads in the local coordinate system is $\{\bar{r}\}^T = \left[F_{\bar{x},i}\ F_{\bar{y},i}\ F_{\bar{z},i}\ M_{\bar{x},i}\ M_{\bar{y},i}\ M_{\bar{z},i}\right]$, where $\{\bar{r}\}$ is also 1x24 и $i = 1,2,3,4$. The introduction of $\theta_{\bar{z},i}$ should not be change the state of stress and that's why the condition

$$M_{i,\bar{z}} = 0.\theta_{\bar{z},i}\,. \tag{7.9}$$

must be met.

The described shell element is cinematically incompatible, but despite this fact it is comfortable for work and with a denser mesh the solution is similar.

7.3.2. Forming of the stiffness matrix of the elements

The stiffness matrix is obtained through the combination of the stiffness matrix of a plane 4-noded element with 8 degrees of freedom $\{k_м\}$ and a slab element with 12 degrees of freedom $\{k_{ог}\}$. For this reason the components of the two matrices are lined in accordance with the nodal parameters of the shell element in the local coordinate system. Because $\{k_м\}$ consists of submatrices 2x2, and $\{k_{ог}\}$ of submatrices 3x3, the correlation between nodal forces in i^{mu} node and the nodal parameters in j^{mu} node is

$$\begin{Bmatrix} F_{\bar{x},i} \\ F_{\bar{y},i} \\ \\ F_{\bar{z},i} \\ M_{\bar{x},i} \\ M_{\bar{y},i} \\ \\ M_{\bar{z},i} \end{Bmatrix} = \begin{bmatrix} \underset{2x2}{[k_{ij,м}]} & \underset{2x3}{[0]} & 0 \\ & & 0 \\ . \ . \ . & . \ . \ . & . \\ \underset{3x2}{[0]} & \underset{3x3}{[k_{ij,ог}]} & 0 \\ & & 0 \\ & & 0 \\ . \ . \ . & . \ . \ . & . \\ 0\,0 & 0\,0\,0 & 0 \end{bmatrix} \begin{Bmatrix} \bar{u}_j \\ \bar{v}_j \\ . \ . \\ \bar{w}_j \\ \theta_{\bar{x},i} \\ \theta_{\bar{y},j} \\ . \ . \\ \theta_{\bar{z},j} \end{Bmatrix}, \tag{7.10}$$

or

$$\{r_i\} = \lfloor k_{ij} \rfloor \{d_j\}. \tag{7.11}$$

The matrix $[k]$ is enlarged to 6x6 with estimation of (7.9). for the reading of the reaction between the elements under different angle is necessary to be made a matrix $[k]$ for each element, that corresponds to the nodal parameters $\{d_i\}^T = \left[u_i\ v_i\ w_i\ \theta_{x,i}\ \theta_{y,i}\ \theta_{z,i}\right]$, i =1,2,3,4 in the global coordinate system. For this reason a matrix from the cosines between the axis of the local and global coordinate system is used

$$[\lambda] = \begin{bmatrix} \lambda_{\bar{x}x} & \lambda_{\bar{x}y} & \lambda_{\bar{x}z} \\ \lambda_{\bar{y}x} & \lambda_{\bar{y}y} & \lambda_{\bar{y}z} \\ \lambda_{\bar{z}x} & \lambda_{\bar{z}y} & \lambda_{\bar{z}z} \end{bmatrix}, \tag{7.12}$$

where $\lambda_{\bar{x}x} = \cos(\bar{x}x)$ and so on. From the matrix $[\lambda]$ is formed the nodal transformable matrix

$$[t] = \begin{bmatrix} [\lambda] & [0] \\ [0] & [\lambda] \end{bmatrix}_{6x6} \qquad .(7.13)$$

The transformable matrix of the whole shell element is

$$[T] = \begin{bmatrix} [t] & [0] & [0] & [0] \\ [0] & [t] & [0] & [0] \\ [0] & [0] & [t] & [0] \\ [0] & [0] & [0] & [t] \end{bmatrix}. \qquad (7.14)$$

Now it can be written

$$\{\bar{d}\} = [T]\{d\} \qquad (7.15)$$

and $$\{\bar{r}\} = [T]\{r\}. \qquad (7.16)$$

The stiffness matrix in the global coordinate system is

$$[k] = [T]^T [\bar{k}][T]. \qquad (7.17)$$

Each submatrix of $[k]$ $\lfloor k_{ij} \rfloor$ is calculated from

$$[k_{ij}] = [t]^T [\bar{k}_{ij}][t] \ , i, j = 1,2,3,4 . \qquad (7.18)$$

7.4. Sloping shells

The sloping shells are shells that have small arrow *f* in comparison to the sizes of its main projection (Fig. 7.11).

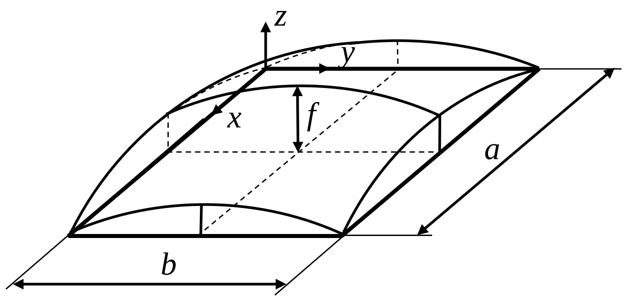


Fig. 7.11

For such shells are met the requirements

$$\frac{f}{a} \leq \frac{1}{5}, \qquad (7.19)$$

where *f* is the arrow and besides that $ds_\alpha \approx dx \;\; u \;\; ds_\beta \approx dy$ (Fig. 7.12).

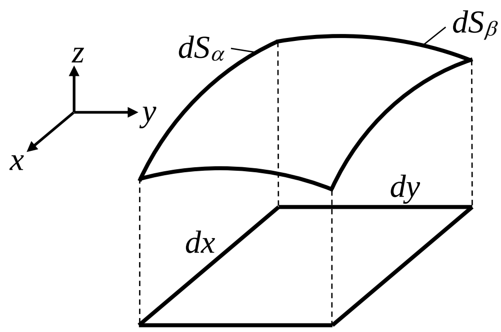


Fig. 7.12

If it is presumed that the middle plane of the sloping shell has equation

$$z(x,y)=a_1+a_2x+a_3y+a_4x^2+a_5xy+a_6y^2 \tag{7.20}$$

with constant curvature

$$\chi_x=\frac{\partial^2 z}{\partial x^2}\ ,\ \chi_y=\frac{\partial^2 z}{\partial y^2}\ ,\ \chi_{xy}=\frac{\partial^2 z}{\partial x\partial y}\ , \tag{7.21}$$

then the transfer of an arbitrary point from the middle plane will be estimated as in the cases with the slabs, from 3 independent functions - $u(x,y), v(x,y)$ и $w(x,y)$.

7.4.1. Calculations and deformations of the sloping shells

On the Fig.7.13 and 7.14 are shown the linear and the angular displacement in the element of the sloping shell of membrane forces, on the basis of which the deformations are determined as well. With *ab* is shown the undeformed state, and with *a/b/* - the deformed state of the element.

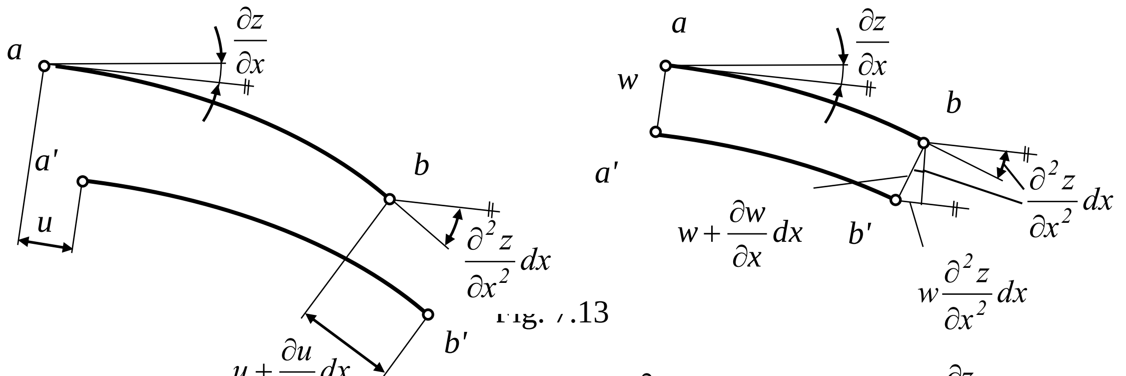


Fig. 7.13

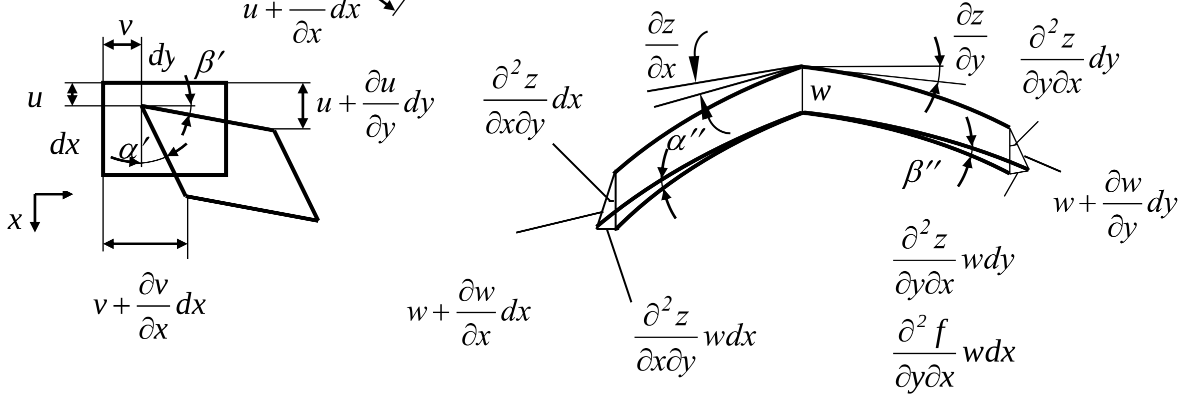


Fig. 7.14

Finally for the deformation can be written

$$\varepsilon_{x,м}=\frac{\partial u}{\partial x}-w\frac{\partial^2 z}{\partial x^2}\ ,\ \varepsilon_{y,м}=\frac{\partial v}{\partial y}-w\frac{\partial^2 z}{\partial y^2}\ ,\ \gamma_{x,мy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2w\frac{\partial^2 z}{\partial x\partial y}\ . \tag{7.22}$$

in constant to the deformation of the plane element, here are obtained addends depending on the curvatures of the middle plane around the point.

According to the theory of the sloping shells during bending of the shell an arbitrary point of the middle plane has three independent displacements, components of the full displacement

$$u = u(x,y)\,,\quad v = v(x,y)\,,\quad w = w(x,y)\,. \tag{7.23}$$

These displacements are defined in the same way as in the slab element, so for the deformation from the bending of the shell element can be written

$$\varepsilon_{x,\text{ог}} = -z\frac{\partial^2 w}{\partial x^2} = \text{-}zk_x\;\;,\;\varepsilon_{y,\text{ог}} = --z\frac{\partial^2 w}{\partial y^2} = \text{-}zk_y\;\;,\;\gamma_{xy,\text{ог}} = -2z\frac{\partial^2 w}{\partial x \partial y} = -zk_{xy}\,. \tag{7.24}$$

Using the combination of the state of stress of the membrane and the bending loads, for the deformation in the shell element can be written

$$\begin{aligned}
\varepsilon_x &= \varepsilon_{x,\text{ог}} + \varepsilon_{x,\text{м}} = -zk_x + \frac{\partial u}{\partial x} - w\chi_x \\
\varepsilon_y &= \varepsilon_{y,\text{ог}} + \varepsilon_{y,\text{м}} = -zk_y + \frac{\partial u}{\partial y} - w\chi_y \\
\gamma_{xy} &= \gamma_{xy,\text{ог}} + \gamma_{xy,\text{м}} = \frac{\partial f}{\partial y} + \frac{\partial u}{\partial x} - 2zk_{xy} - 2w\chi_{xy}\,.
\end{aligned} \tag{7.25}$$

7.4.2. Approximation of the displacements of the element

As it was noted, according to the theory of the sloping shell the displacement of an arbitrary point of the middle plane is defined with three independent functions - $u = u(x,y)$, $v = v(x,y)$, $w = w(x,y)$. The calculation of the approximation of the functions is done with polynomials as in the 2D and slab elements.

7.4.3. Correlation between the stresses and the deformations

They are given by the general law of Hooke $\{\sigma\} = [E]\{\varepsilon\}$. The stresses can be obtained through superposition as well, for example for the stress σ_x can be written

$$\sigma_x = \sigma_{x,\text{ог}} + \sigma_{x,\text{м}} = \frac{M_x z}{t^3/12} + \frac{N_x}{t}\;\;. \tag{7.26}$$

7.5. Axially symmetry shells

The axially symmetrical shell is a body, which is obtained during the rotation of a curved line around an axis. Under arbitrary conditions of stress and support of the shell, a point of the middle plane has three displacements, shown in Fig. 7.15.

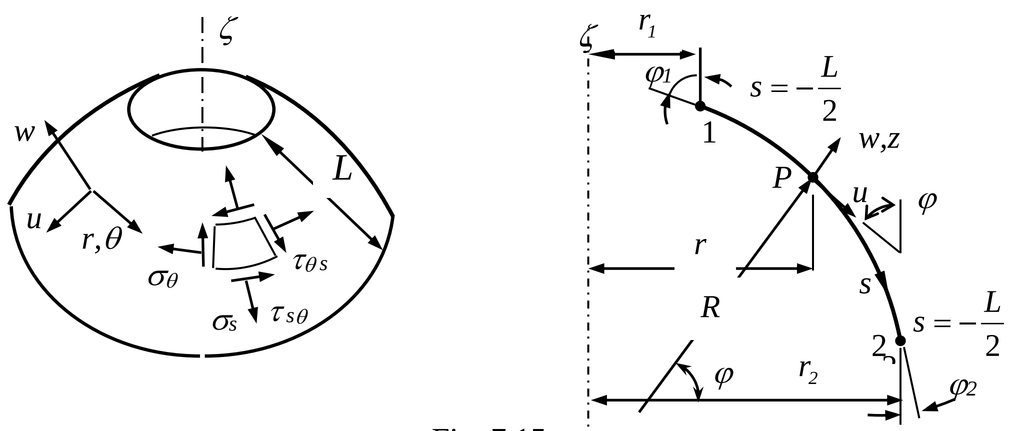


Fig. 7.15

The geometrical dependencies are:

$$R_\theta = \frac{r}{\cos\varphi} \quad , \quad R_s = -\frac{ds}{d\varphi} \quad , \quad \sin\varphi = \frac{dr}{ds} \quad , \quad \cos\varphi = \frac{d\zeta}{ds} . \tag{7.27}$$

R_θ и R_s are the main radii of curvature of the point and their centers are positioned along the normal vector to the middle plane in the point. The center of the radius R_θ lays on the axis ξ.

If the characteristics of the material, supports and the loading are independent from θ, the displacement and the stress are also independent from it, and so

$$v = \tau_{s\theta} = 0 . \tag{7.28}$$

On the basis of the change of the radius r according to Fig.7.6, a and 7.6, b, for the relative deformation ε_θ can be written

$$\varepsilon_\theta = \frac{2\pi\left[r + \frac{\sin\varphi\, u\left(R_s + z\right)}{R_s} + w\cos\varphi - z\frac{\partial w}{\partial s}\sin\varphi \right] - 2\pi r}{2\pi r} = \\ = \frac{u\sin\varphi + w\cos\varphi}{r} + z\frac{\sin\varphi}{r}\left(\frac{u}{R_s} - \frac{\partial w}{\partial s} \right) \tag{7.29}$$

Taking into account the dependence (7.5), for the deformation can be written

$$\begin{array}{ll} \varepsilon_s = \varepsilon_{s,м} + \varepsilon_{s,oг}, & \varepsilon_{s,oг} = zk_s \\ \varepsilon_\theta = \varepsilon_{\theta,м} + \varepsilon_{\theta,oг}, & \varepsilon_{\theta,oг} = zk_\theta \end{array} . \tag{7.30}$$

where

$$\begin{array}{l} \varepsilon_{s,м} = \frac{du}{ds} + \frac{w}{R_s} \quad , \quad \varepsilon_{\theta,м} = \frac{u\sin\varphi + w\cos\varphi}{r} \\ k_s = \frac{d}{ds}\left(\frac{u}{R_s} \right) - \frac{d^2 w}{ds^2} \quad , \quad k_\theta = \frac{\sin\varphi}{r}\left(\frac{u}{R_s} - \frac{dw}{ds} \right) \end{array} , \tag{7.31}$$

where $\varepsilon_{s,м}$ и $\varepsilon_{\theta,м}$ are membrane deformations of the middle plane and k_s и k_θ the changes of the curvatures of the middle plane. The vector of the displacements is

$$\{\varepsilon\}^T = \left[\varepsilon_{s,м} \ \varepsilon_{\theta,м} \ -zk_s \ -zk_\theta \right]. \tag{7.32}$$

Finite element for modeling for such shell is shown in Fig. 7.16.

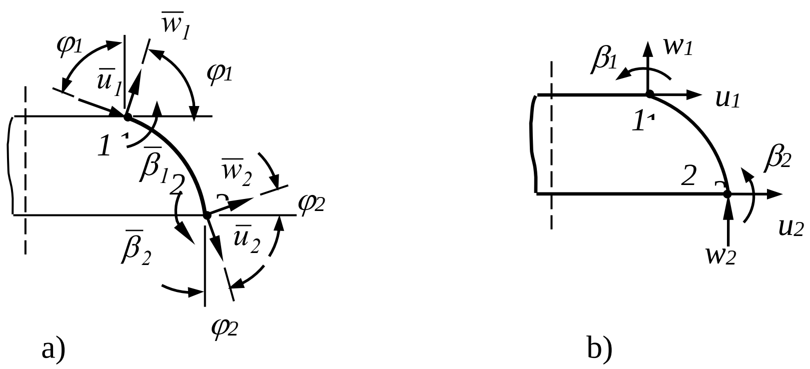


Fig. 7.16

The transformation of the nodal parameters from the local (Fig. 7.16, a) to the global coordinate system (Fig. 7.16, b) for common node is done according the dependencies

$$\begin{Bmatrix} \overline{u}_i \\ \overline{w}_i \\ \overline{\beta}_i \end{Bmatrix} = \begin{bmatrix} sin\,\varphi_i & -cos\,\varphi_i & 0 \\ cos\,\varphi_i & sin\,\varphi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ w_i \\ \beta_i \end{Bmatrix}, \tag{7.31}$$

where $\overline{u}_i$,$\overline{w}_i$ *и* $\overline{\beta}_i$ care the parameters in the local coordinate system, and u_i, v_i и β_i are the parameters in the global one.

The change of the strain energy of deformation is defined from

$$\delta U = \int_{-L/2}^{L/2} \{\delta\varepsilon\}^T \begin{bmatrix} [E_м] & [0] \\ [0] & [E_{ог}] \end{bmatrix} \{\varepsilon\} 2\pi r ds\,, \tag{7.32}$$

where

$$[E_м] = \frac{Et}{1-\mu^{12}} \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix}\,,\ [E_{ог}] = \frac{Et^3}{12\left(1-\mu^2\right)}. \tag{7.33}$$

After the introduction of approximating function for displacement can be obtained the matrices $[N], [B]$ и $[k]$.

The membrane forces in the shell element are determined from

$$\begin{Bmatrix} N_s \\ N_\theta \end{Bmatrix} = \frac{Et}{1-\mu^2} \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{s,м} \\ \varepsilon_{\theta,ог} \end{Bmatrix}, \tag{7.34}$$

and in the bending moments from

$$\begin{Bmatrix} M_s \\ M_\theta \end{Bmatrix} = \frac{Et^3}{12\left(1-\mu^2\right)} \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix} \begin{Bmatrix} k_s \\ k_\theta \end{Bmatrix}. \tag{7.35}$$

If $\varphi = const$ a shell element is obtained, shown in Fig. 7.17.

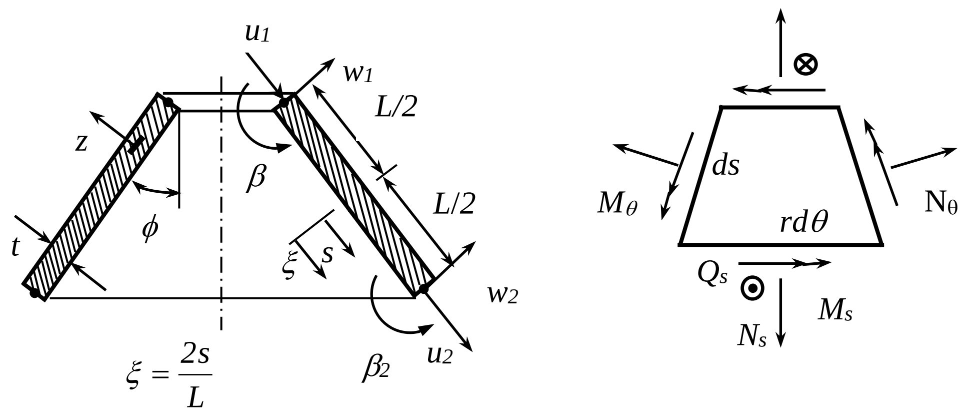


Fig. 7.17

With such an element the construction can be regarded as a collection of conic shells. In this case the vector of the deformation is determined as follows

$$\{\varepsilon\} = \begin{Bmatrix} \dfrac{du}{ds} \\ \dfrac{w\cos\varphi + r\sin\varphi}{r} \\ -z\dfrac{d^2w}{ds^2} \\ -z\dfrac{\sin\varphi}{r}\dfrac{dw}{ds} \end{Bmatrix}. \tag{7.36}$$

If it is accepted that *u* is a linear function of *s*, and *w* is a polynomial of third degree, than it can be written

$$\begin{aligned} u &= a_1 + a_2 s \\ w &= a_3 + a_4 s + a_5 s^2 + a_6 s^3 \end{aligned} \quad .(7.37)$$

The nodal parameters and the nodal coordinates after which the matrices $[N]$, $[B]$ и $[k]$ are constructed define the polynomial coefficients.