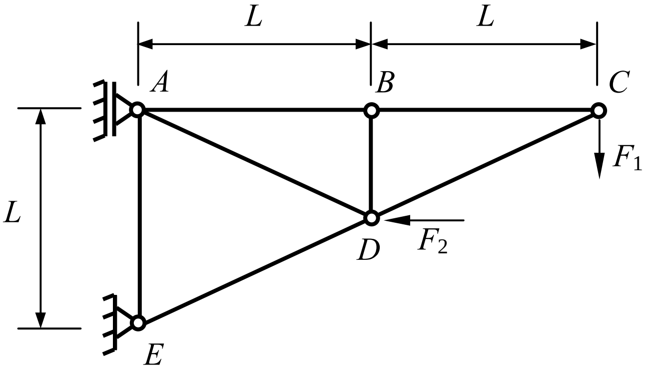
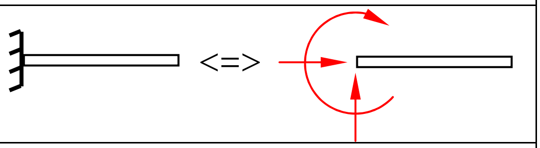
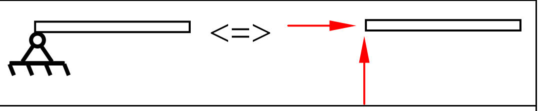
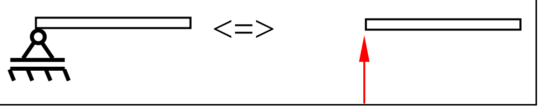
For the given truss construction determine:

1. The support reactions;
2. The internal forces in all elements.

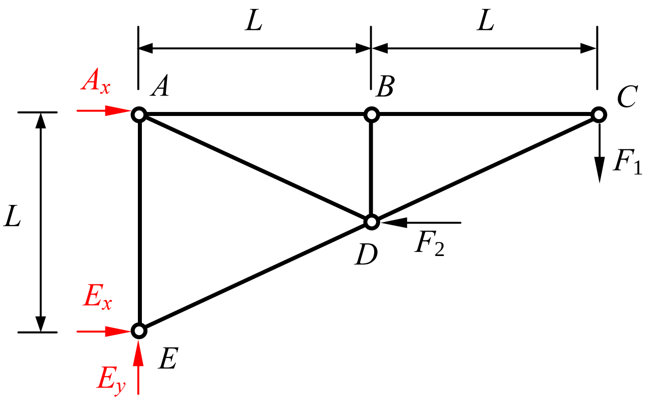
The members *AB*, *BC*, *CD*, *DE* and *AE* are rods and points *A*, *B*, *C*, *D* and *E* are joints.
F_1= 30 kN, F_2= 20 kN, L=2 m.



The first step of the solution is related to drawing the Free Body Diagram (FBD). That means that we have to redraw the geometry of the construction together with the external forces given in the condition, after which we replace the supports with support reactions. The types of supports which are used in Statics for 2D problems are given in the table below:

Kind of supporting	**Number of reactions**	**Scheme**
Fixed support	3	
Pin support	2	
Roller support	1	

The FBD of our construction is:



Applying the equilibriums we can find the support reactions

$$\sum F_x = 0 \;\Rightarrow\; A_x + E_x - F_2 = 0 \;\Rightarrow\; A_x + E_x = F_2;$$

$$\sum F_y = 0 \;\Rightarrow\; E_y - F_1 = 0 \;\Rightarrow\; E_y = F_1 \;\Rightarrow\; \boxed{E_y = 30 \text{ kN}}.$$

The third equilibrium can be written with respect to point A. We need to determine the distance between F_2 and point A first. From the similarity of $\triangle ACE$ and $\triangle BCD$ it can be easily concluded that $BD = \frac{L}{2}$.

The third equilibrium leads to the following:

$$\sum M_{z_A} = 0 \;\Rightarrow\; E_x.L - F_1.2L - F_2.\frac{L}{2} = 0 \;\Rightarrow\; E_x = 2.F_1 + 0{,}5.F_2 = 60+10 \;\Rightarrow\; \boxed{E_x = 70\text{ kN}}.$$

Substituting E_x in the first equilibrium we can determine A_x:

$$A_x = F_2 - E_x = 20 - 70 \;\Rightarrow\; \boxed{A_x = -50\text{ kN}}.$$

The reaction E_y does not take place in the momentum equilibrium since the distance between E_y and point A is zero. **Under distance between a force and a point one should understand the distance between the line over which the force is lying and the point.**

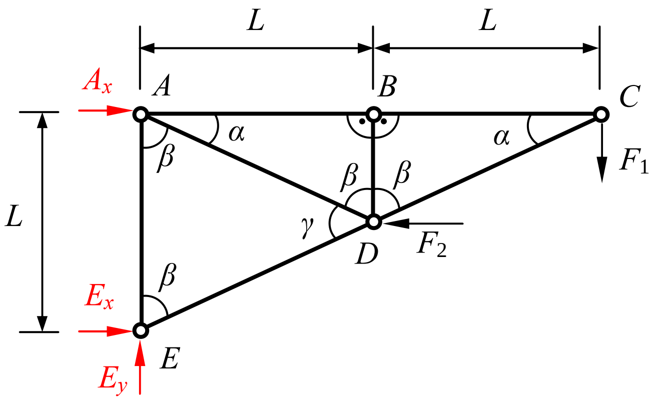
The negative value of the reaction A_x means that its direction should be opposite to the one we have chosen.

Before moving to the next step of the solution we must determine the angles which the members of the construction form between each other. Since the triangles $\triangle BCD$ and $\triangle ABD$ are equal, we can write:

$\measuredangle BCD = \measuredangle BAD = \alpha$;

$\measuredangle BDC = \measuredangle BDA = \measuredangle DAE = \measuredangle DEA = \beta$;

$\measuredangle ADE = \gamma$.



Considering $\triangle AEC$ we can write:

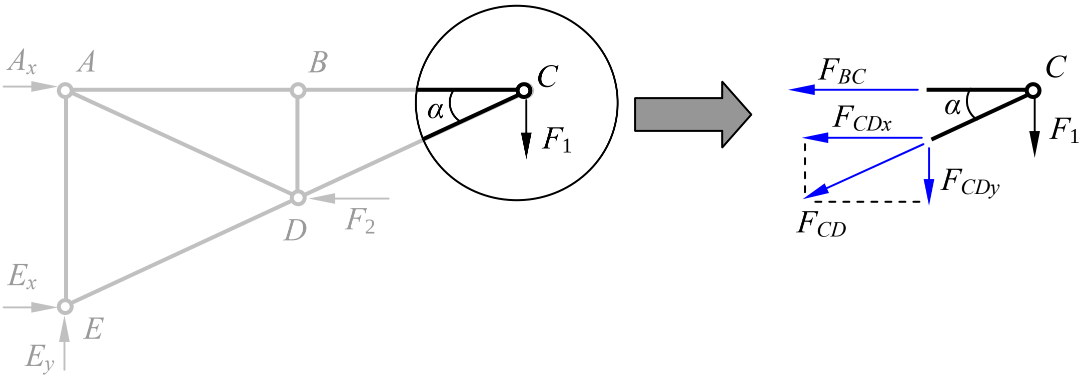
$$\frac{L}{2L} = tg(\alpha) \;\Rightarrow\; tg(\alpha) = 0{,}5 \;\Rightarrow\; \boxed{\alpha = arctg(0{,}5) = 26{,}6^\circ}.$$

And since $\triangle AEC$ has a 90° angle $\Rightarrow$ $\beta = 90^\circ - \alpha = 90^\circ - 26{,}6^\circ \;\Rightarrow\; \boxed{\beta = 63{,}4^\circ}$.

Regarding $\triangle AED \Rightarrow \gamma = 180^\circ - 2\beta = 180^\circ - 126{,}8^\circ \Rightarrow \boxed{\gamma = 53{,}2^\circ}$.

In order to find the internal forces, which is the task in the second stage of the problem, we have to isolate joints. **The solution can begin by isolating a joint in which there are maximum 2 trusses crossing each other**. That means that we can begin either with joint C or E but **not** with joints A, B or D.

We will isolate joint C first. For that purpose we draw a circle around joint C and we remove everything outside the circle. We replace the part of the construction which we have removed with internal forces which **directions should be opposite to the joint** as it is shown below.



The forces F_{CD} and F_{BC} from the right figure must have such values so that this isolated part behaves in the same way as it behaves when it is a part of the whole construction.

The force F_{CD} is resolved into its horizontal and vertical component F_{CDx} and F_{CDy} respectively. For the isolated joint we apply the following equilibriums:

$$\sum F_x = 0 \quad \Rightarrow \quad F_{BC} + F_{CDx} = 0\,;$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{CDy} + F_1 = 0 \quad \Rightarrow \quad F_{CDy} = -F_1 \quad \Rightarrow \quad F_{CDy} = -30\ \text{kN}\,.$$

Knowing the value of the vertical component F_{CDy} we can find F_{CD} as follows:

$$F_{CDy} = F_{CD}.\sin(\alpha) \quad \Rightarrow \quad F_{CD} = \frac{F_{CDy}}{\sin(26{,}6^\circ)} \quad \Rightarrow \quad F_{CD} = \frac{-30}{0{,}448} \quad \Rightarrow \quad \boxed{F_{CD} = -67\ \text{kN}}\,.$$

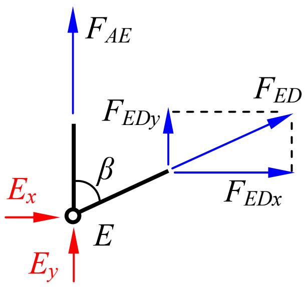
Knowing F_{CD} we can find F_{CDx}

$$F_{CDx} = F_{CD}.\cos(\alpha) \quad \Rightarrow \quad F_{CDx} = (-67).0{,}894 \quad \Rightarrow \quad F_{CDx} = -59{,}9\ \text{kN}$$

which value can be used for determination of F_{BC}

$$F_{BC} = -F_{CDx} \quad \Rightarrow \quad F_{BC} = -(-59{,}9) \quad \Rightarrow \quad \boxed{F_{BC} = 59{,}9\ \text{kN}}$$

Next we isolate joint E.



We use the same equilibriums and we apply the same procedure to find the internal forces.

$$\sum F_x = 0 \quad \Rightarrow \quad E_x + F_{EDx} = 0 \quad \Rightarrow \quad F_{EDx} = -E_x \quad \Rightarrow \quad F_{EDx} = -70\ \text{kN}\,;$$

$$F_{EDx} = F_{ED}.\sin(\beta) \quad \Rightarrow \quad F_{ED} = \frac{F_{EDx}}{\sin(63{,}4^\circ)} \quad \Rightarrow \quad F_{ED} = \frac{-70}{0{,}894} \quad \Rightarrow \quad \boxed{F_{ED} = -78{,}3\ \text{kN}}$$

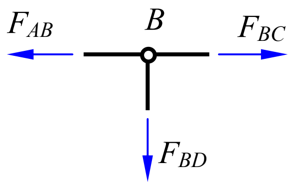
$$\sum F_y = 0 \quad \Rightarrow \quad F_{AE} + E_y + F_{EDy} = 0 \quad \Rightarrow \quad F_{AE} = -E_y - F_{EDy}\,.$$

Since we know the value of F_{ED} we can determine F_{EDy}.

$$F_{EDy} = F_{ED}.\cos(\beta) \quad \Rightarrow \quad F_{EDy} = -78{,}3.\cos(63{,}4^\circ) = -78{,}3.0.448 \quad \Rightarrow \quad F_{EDy} = -35{,}1\ \text{kN}$$

$$\Rightarrow \quad F_{AE} = -E_y - F_{EDy} = -30 - (-35{,}1) \quad \Rightarrow \quad \boxed{F_{AE} = 5{,}1\ \text{kN}}$$

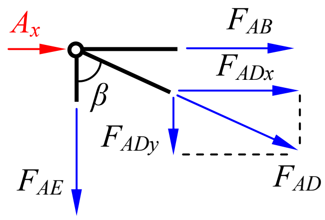
We isolate joint *B*.



$$\sum F_x = 0 \quad \Rightarrow \quad F_{AB} - F_{BC} = 0 \quad \Rightarrow \quad F_{AB} = F_{BC} \quad \Rightarrow \quad \boxed{F_{AB} = 59{,}9 \text{ kN}}$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{BD} = 0$$

The last internal force to be found is F_{AD}. Its value can be determined by isolating joint *A*.



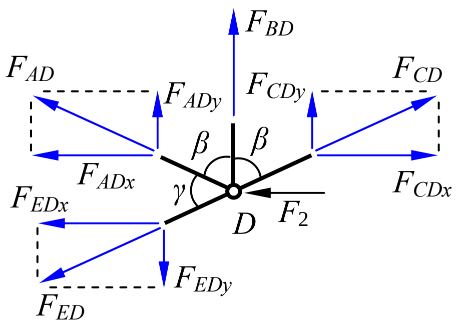
$$\sum F_x = 0 \quad \Rightarrow \quad F_{AB} + F_{ADx} + A_x = 0 \quad \Rightarrow \quad F_{ADx} = -A_x - F_{AB} = -(-50) - 59{,}9 \quad \Rightarrow \quad F_{ADx} = -9{,}9 \text{ kN}$$

$$F_{ADx} = F_{AD}.\sin(\beta) \quad \Rightarrow \quad F_{AD} = \frac{F_{ADx}}{\sin(63{,}4^{\circ})} \quad \Rightarrow \quad F_{AD} = \frac{-9{,}9}{0{,}894} \quad \Rightarrow \quad \boxed{F_{AD} = -11{,}1 \text{ kN}}$$

$$F_{ADy} = F_{AD}.\cos(\beta) \quad \Rightarrow \quad F_{ADy} = -11{,}1.\cos(63{,}4^{\circ}) \quad \Rightarrow \quad F_{ADy} = -11{,}1.0{,}448 \quad \Rightarrow \quad F_{ADy} = -5 \text{ kN}$$

CHECK

In order to check our solution we have to isolate the only joint which is not isolated up to now. This is joint *D*. By applying the equilibriums we have to check out whether the forces in this joint balance each other, i.e. their sum is zero.



$$\sum F_x = 0 \quad \Rightarrow \quad F_{EDx} + F_{ADx} + F_2 - F_{CDx} = 0 \quad \Rightarrow \quad -70 - 9{,}9 + 20 - (-59{,}9) = 0$$

$$\Rightarrow \quad -79{,}9 + 79{,}9 = 0 \quad \Rightarrow \quad \boxed{0 = 0}$$

$$\sum F_y = 0 \quad \Rightarrow \quad -F_{EDy} + F_{ADy} + F_{BD} + F_{CDy} = 0 \quad \Rightarrow \quad -(-35{,}1) - 5 + 0 - 30 = 0$$

$$\Rightarrow \quad 35{,}1 - 35 = 0 \quad \Rightarrow \quad \boxed{0 \approx 0}$$

In the last equilibrium a small difference 0,1 appears but it is due to rounding in some calculations.