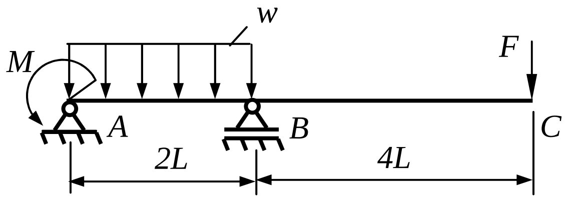
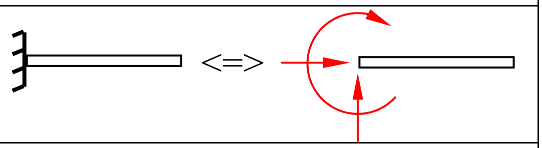
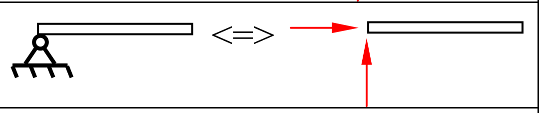
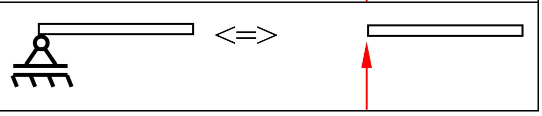
For the given beam construction determine:

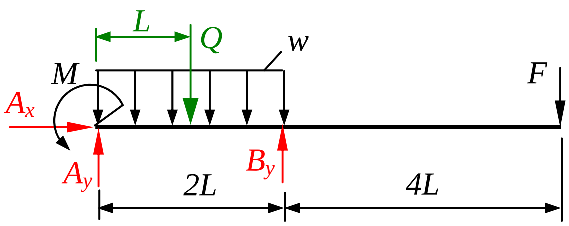
1. the support reactions;
2. the internal forces and draw their diagrams.

$F = 2$ kN, $M = 1$ kN.m, $w = 30$ kN/m, L=0,5 m.



In the first step of the solution the Free Body Diagram (FBD) has to be drawn. That means that we have to redraw the geometry of the construction together with the external forces given in the condition, after which we replace the supports with support reactions. The types of supports which are used for 2D problems are given in the table below:

Kind of supporting	**Number of reactions**	**Scheme**
Fixed support	3	
Pin support	2	
Roller support	1	



By applying the equilibriums we can find the support reactions. In order to account for the distributed load w it has to be represented as a concentrated load. Since this is a **rectangular load we concentrate it in the middle of the length over which it is distributed and we determine its value by multiplying the intensity by this length**:

$Q = w.2L = 30.2.0,5 = 30$ kN

Now for the support reactions we can write:

$\sum F_x = 0 \;\Rightarrow\; \boxed{A_x = 0}$;

$\sum F_y = 0 \;\Rightarrow\; A_y - Q + B_y - F = 0 \;\Rightarrow\; A_y + B_y = F + Q \;\Rightarrow\; A_y + B_y = 32$.

The third equilibrium can be written with respect to point B as for instance.

$\sum M_{z_B} = 0 \;\Rightarrow\; -M + A_y.2L - Q.L + F.4L = 0 \;\Rightarrow\; A_y.2L = M + Q.L - F.4L$

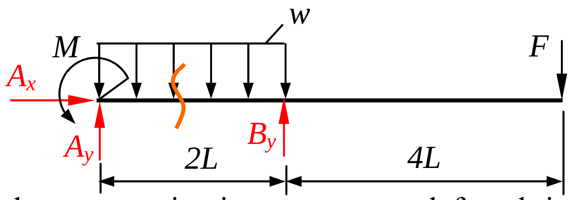
$$\Rightarrow\; A_y = \frac{M}{2L} + \frac{Q}{2} - F.2 \;\Rightarrow\; A_y = \frac{1}{2.0,5} + \frac{30}{2} - 2.2 \;\Rightarrow\; \boxed{A_y = 12 \text{ kN}}$$

Referring to the second equilibrium we can determine B_y:

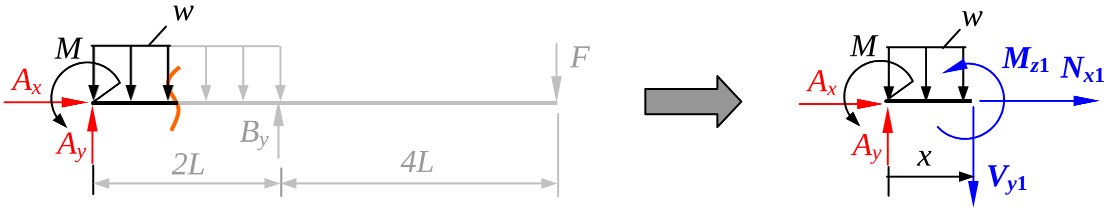
$B_y = 32 - A_y \Rightarrow B_y = 32 - 12 \Rightarrow \boxed{B_y = 20 \text{ kN}}$

The next step is to determine the internal forces. For that purpose we have to make sections. **Each concentrated force and moment (including reactions) have to be regarded as a boundary between two sections. The beginning or the end of each distributed load is also a boundary between two sections. Representation of a distributed load as a concentrated one - *Q* must not be considered as a boundary.** In the construction regarded here we need to make 2 sections – the first one between points *A* and *B*; the second one between points *B* and *C*.

We will begin by making a section between points *A* and *B*.



This section divides the construction into two parts – left and right one. Let us now remove the right part, i.e. we remove everything which stands to the right of the section and we substitute this with internal forces as it is shown on the picture below.



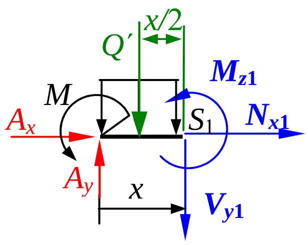
The internal forces (in blue color) are 3 - a horizontal (normal), a vertical (shearing) force and a bending moment and they have standard directions. For a left section (in this case we have kept everything to the left of the section) these directions are shown in the drawing above. Since this is the first section we put an index 1 for each internal force.

The position of the section is determined by a coordinate *x* which origin is selected to be at point *A*. Since this section is made **somewhere** between points *A* and *B* the current coordinate *x* can change in the following range:

$0 \le x \le 2L$ (***x* = 0** at point ***A***; ***x* = 2*L*** at point ***B***).

Before applying the equilibriums to find the internal forces, we have to concentrate the **part** of the distributed load *w* which has left after removing everything to the right of the section:

$Q' = w.x$



$0 \le x \le 2L$

Now we can write the equilibriums

$\sum F_x = 0 \Rightarrow A_x + N_{x1} = 0 \Rightarrow \boxed{N_{x1} = 0}$;

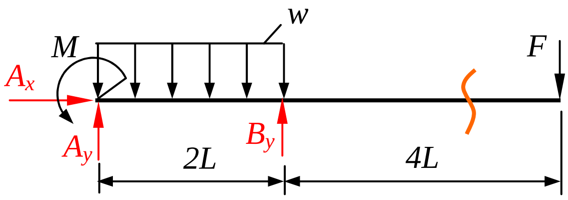
$\sum F_y = 0 \Rightarrow A_y - Q' - V_{y1} = 0 \Rightarrow V_{y1} = A_y - Q' \Rightarrow \boxed{V_{y1} = 12 - 30.x}$.

The third equilibrium is recommended to be written with respect to the point of the section which is assigned with S_1.

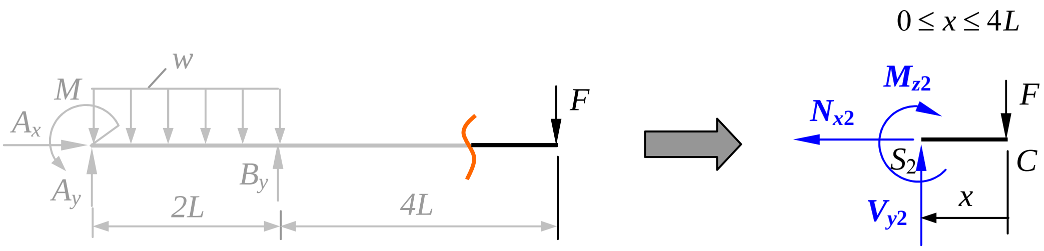
$$\sum M_{z_{S_1}} = 0 \quad \Rightarrow \quad -M + A_y.x - Q'.\frac{x}{2} - M_{z1} = 0 \quad \Rightarrow \quad M_{z1} = -M + A_y.x - Q'.\frac{x}{2}$$

$$\Rightarrow \quad \boxed{M_{z1} = -1 + 12.x - 15x^2}$$

The second section is between points B and C.



This section also divides the construction into two parts – left and right one. This time we will remove everything which stands to the left of the section and we will substitute this again with internal forces (of course, we can remove again everything which is to the right of the section but in this way the equations of the internal forces will be more complicated).



The internal forces are again 3 (in blue color) but since we regard a right section now (we have removed everything to the left), their directions are opposite to the ones for a left section. We use an index 2 for each internal force to denote that it is related to the second section.

We can select that the current coordinate x for the second section starts from point C and goes to the left. We have to mind only that

$0 \le x \le 4L$ (**$x = 0$** at point **C**; **$x = 4L$** at point **B**).

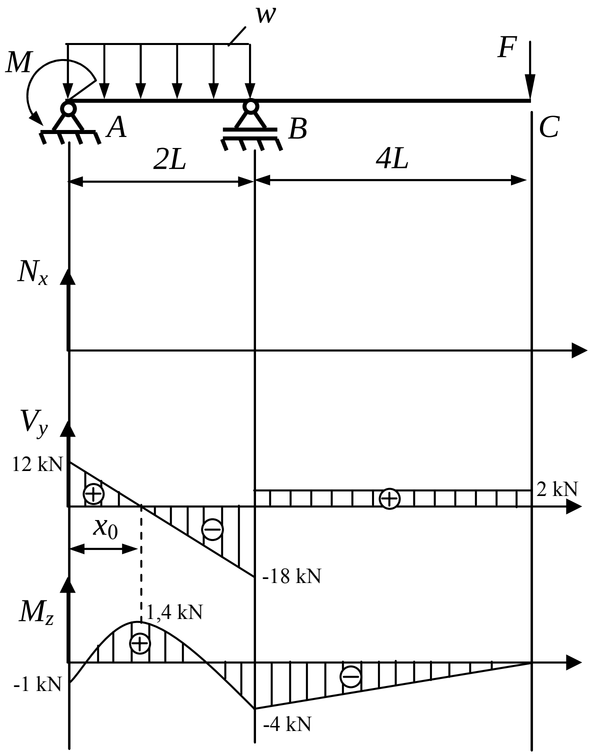
Now we write the equilibriums

$$\sum F_x = 0 \quad \Rightarrow \quad \boxed{N_{x2} = 0};$$

$$\sum F_y = 0 \quad \Rightarrow \quad F - V_{y2} = 0 \quad \Rightarrow \quad V_{y2} = F \quad \Rightarrow \quad \boxed{V_{y2} = 2\ \text{kN}}.$$

The third equilibrium is also recommended to be written with respect to the point of the section S_2.

$$\sum M_{z_{S_2}} = 0 \quad \Rightarrow \quad M_{z2} + F.x = 0 \quad \Rightarrow \quad \boxed{M_{z2} = -2.x}$$



Now when we have the equations of the internal forces in the two sections we can draw their diagrams. We need to draw again the construction and leave some space below for the diagrams.

Since N_x is zero in both sections there will be no diagram for this internal force.

We regard the equation of V_{y1}. This is a linear function. Its diagram can be drawn by two points. Since $0 \le x \le 2L$ we can calculate the value of V_{y1} for $x = 0$ (at point A) and for $x = 2L$ (at point B):

for $x = 0 \;\Rightarrow\; V_{y1} = 12 - 30.0$

$\Rightarrow\; V_{y1} = 12 \text{ kN}$;

for $x = 2L \;\Rightarrow\; V_{y1} = 12 - 30.2.0,5$

$\Rightarrow\; V_{y1} = -18 \text{ kN}$.

On the diagram for V_y we draw value 12 kN at point A and -18 kN at point B and we join the two values with a straight line (since V_{y1} is a linear function).

For the second section V_{y2} is a constant which diagram is a line parallel to the horizontal axis.

The expression for M_{z1} is a square function which means that its diagram will be a parabolic curve. We calculate the value of M_{z1} at point A and B:

for $x = 0 \;\Rightarrow\; M_{z1} = -1 + 12.0 - 15.0 \;\Rightarrow\; M_{z1} = -1 \text{ kN.m}$;

for $x = 2L \;\Rightarrow\; M_{z1} = -1 + 12.2.0,5 - 15.(2.0,5)^2 \;\Rightarrow\; M_{z1} = -4 \text{ kN.m}$.

Every time when we notice that the diagram of V_y passes through zero (this is the diagram of V_{y1} in our case) **we have to find at what value of the current coordinate x V_y is zero**. We will assign this distance with x_0. **After finding x_0 we have to calculate M_z for $x = x_0$**. This procedure is done in the following way:

We substitute V_{y1} with zero:

$$V_{y1} = 0 = 12 - 30.x_0 \;\Rightarrow\; 30.x_0 = 12 \;\Rightarrow\; x_0 = \frac{12}{30} \;\Rightarrow\; x_0 = 0,4 \text{ m}.$$

Now we calculate

for $x = x_0 = 0,4 \text{ m} \;\Rightarrow\; M_{z1} = -1 + 12.0,4 - 15.(0,4)^2 \;\Rightarrow\; M_{z1} = 1,4 \text{ kN.m}$.

Through these three points (x = 0, x = 2L and x = x_0) we draw a parabola which is the diagram of M_{z1}.

The expression for M_{z2} is a linear function and we can draw its diagram in a similar way we did for the V_{y1}:

for $x = 0 \;\Rightarrow\; M_{z2} = -2.0 \;\Rightarrow\; M_{z2} = 0 \text{ kN.m}$;

for $x = 4L \;\Rightarrow\; M_{z2} = -2.4.0,5 \;\Rightarrow\; M_{z2} = -4 \text{ kN.m}$.

We just have to consider that, since for the second section the direction of x is opposite to the one in the first section, x = 0 is at point C and x = 4L is at point B.

We connect the two values for M_{z2} with a straight line to draw the diagram.