How to design beam structures loaded in bending

Only 2D beam and truss structures will be regarded blow.

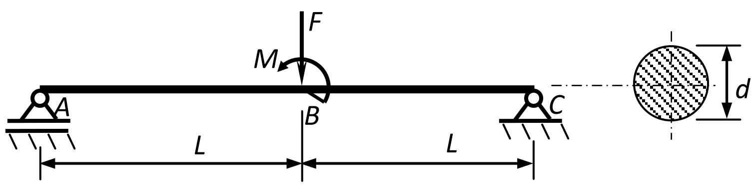
Designing a structure is a task which contains the following steps:

- drawing the Frere Body Diagram (FBD)
- determining the support reactions by using the equilibriums
- applying the method of the section to determine the internal forces by using the equilibriums
- drawing the internal forces diagrams and determine where is the critical section
- applying the design formulas

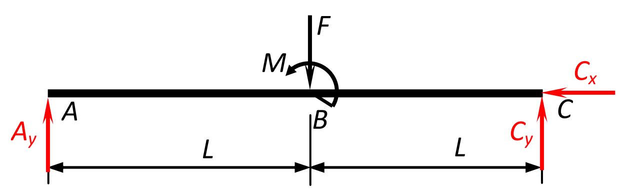
To demonstrate the above described procedure the following two examples will be regarded:

Example 1 – structure with non-standard cross-section

Design the structure if F = 20 kN, M = 4 kN.m, L = 1 m. The cross-section is a circle with diameter d. The material is ductile standard steel S235JR with yielding stress σ_y = 235 MPa. Use a factor of safety FS = 1,2. The internal force V_y is neglected.



1. *FBD*



2. *Determining the support reactions by using the equilibriums*

$$\sum F_x = 0 \;\Rightarrow\; \boxed{C_x = 0 \text{ kN}};$$

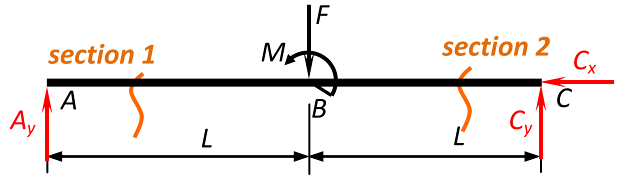
$$\sum F_y = 0 \;\Rightarrow\; A_y + C_y - F = 0 \;\Rightarrow\; A_y + C_y = F.$$

$$\sum M_{z_A} = 0 \;\Rightarrow\; M - F.L + C_y.2L = 0 \;\Rightarrow\; C_y = \frac{F.L}{2.L} - \frac{M}{2.L} \;\Rightarrow\; C_y = 10 - 2 \;\Rightarrow\; \boxed{C_y = 8 \text{ kN.m}}$$

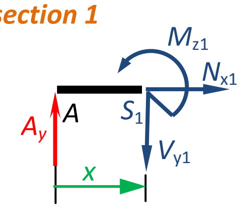
From the second equilibrium $\Rightarrow A_y = F - C_y \;\Rightarrow\; A_y = 20 - 8 \;\Rightarrow\; \boxed{A_y = 12\text{kN}}$

3. *Applying the method of the section to determine the internal forces by using the equilibriums*

Two sections have to be made in the regarded example



For ***section 1*** the left part will be taken (that means everything to the right of this section will be removed). The internal forces are drawn in the point of the section - S_1.



The coordinate x starts from point A and changes to the right. The boundaries are $0 \le x \le L$.

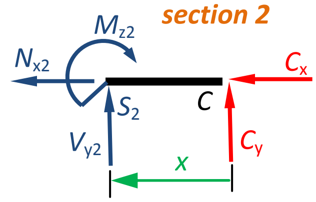
The equilibriums are applied to determine the internal forces.

$$\sum F_x = 0 \;\Rightarrow\; \boxed{N_{x1} = 0 \text{ kN}};$$

$$\sum F_y = 0 \;\Rightarrow\; A_y - V_{y1} = 0 \;\Rightarrow\; V_{y1} = A_y \;\Rightarrow\; \boxed{V_{y1} = 12\text{kN}}.$$

$$\sum M_{z_{S1}} = 0 \;\Rightarrow\; M_{z1} - A_y.x = 0 \;\Rightarrow\; M_{z1} = A_y.x \;\Rightarrow\; \boxed{M_{z1} = 12.x}$$

For ***section 2*** the right part will be taken (everything to the left will be removed). The internal forces are drawn in the point of the section - S_2.



The coordinate x starts from point C and changes to the left. The boundaries are $0 \le x \le L$.

The equilibriums are applied to determine the internal forces.

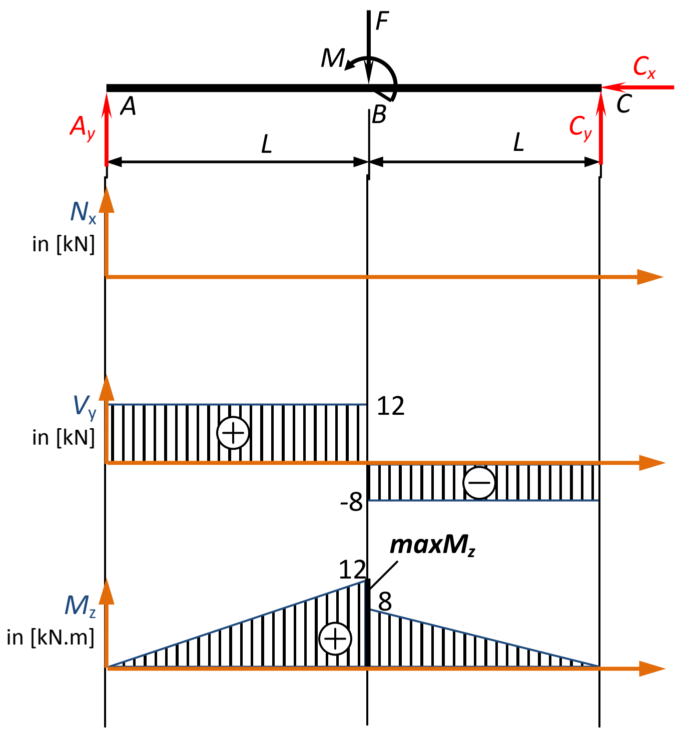
$$\sum F_x = 0 \;\Rightarrow\; N_{x2} + C_x = 0 \;\Rightarrow\; N_{x2} = -C_x \;\Rightarrow\; \boxed{N_{x2} = 0 \text{ kN}};$$

$$\sum F_y = 0 \;\Rightarrow\; C_y + V_{y2} = 0 \;\Rightarrow\; V_{y2} = -C_y \;\Rightarrow\; \boxed{V_{y2} = -8 \text{ kN}};$$

$$\sum M_{z_{S2}} = 0 \;\Rightarrow\; M_{z2} - C_y.x = 0 \;\Rightarrow\; M_{z2} = C_y.x \;\Rightarrow\; \boxed{M_{z2} = 8.x}.$$

4. *Drawing the internal forces diagrams and determining where is the critical section*

Section 1	**Section 2**
$N_{x1} = 0$ kN - constant	$N_{x2} = 0$ kN - constant
$V_{y1} = 12$kN - constant	$V_{y2} = -8$ kN - constant
$M_{z1} = 12.x$ - linear function $0 \le x \le L$ for $x = 0 \quad \Rightarrow \quad M_{z1} = 0$kN.m for $x = L = 1$m $\quad \Rightarrow \quad M_{z1} = 12$kN.m	$M_{z2} = 8.x$ - linear function $0 \le x \le L$ for $x = 0 \quad \Rightarrow \quad M_{z2} = 0$kN.m for $x = L = 1$m $\quad \Rightarrow \quad M_{z2} = 8$kN.m



The internal force V_y is neglected.

Critical cross-section - point *B* with maxM_z = 12 kN.m (Since N_x = 0 and V_y is neglected)

5. *Applying the design formulas*

When making design only M_z is accounted (N_x and V_y are neglected).

The design formula is:

$$\frac{|\max M_z|}{S_z} \le \sigma_{all}$$

where S_z is the section modulus.

The allowable stresses σ_{all} can be calculated by the formula

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{235}{1,2} = 195,8\text{MPa}$$

All units in the design formula must be in N, m, Pa which means that

$$\max M_z = 12\text{kN.m} = 12.10^3\,\text{N.m}$$

$$\sigma_{all} = 195,8\text{MPa} = 195,8.10^6\,\text{Pa}$$

Substituting with the values in the design formula follows

$$\frac{|\max M_z|}{S_z} \le \sigma_{all} \;\Rightarrow\; \frac{12.10^3}{S_z} \le 195,8.10^6 \;\Rightarrow\; \frac{12.10^3}{195,8.10^6} \le S_z \;\Rightarrow\; S_z \ge 0,0613.10^{-3}\,\text{m}^3$$

Since the cross-section is a circle the formula for the section modulus is (*see Strength of Materials Handbook, page 6*)

$$S_z = \frac{\pi.d^3}{32}$$

So it follows

$$S_z = \frac{\pi.d^3}{32} \ge 0,0613.10^{-3}\,\text{m}^3 \;\Rightarrow\; d^3 \ge \frac{32.0,0613.10^{-3}}{3,14} \;\Rightarrow\; d^3 \ge 0,6247.10^{-3}$$

$$\Rightarrow\; d \ge \sqrt[3]{0,6247.10^{-6}} \;\Rightarrow\; d \ge 0,855.10^{-1}\,\text{m} \;\Rightarrow\; d \ge 85,5.10^{-3}\,\text{m} \;\Rightarrow\; d \ge 85,5\text{mm}$$

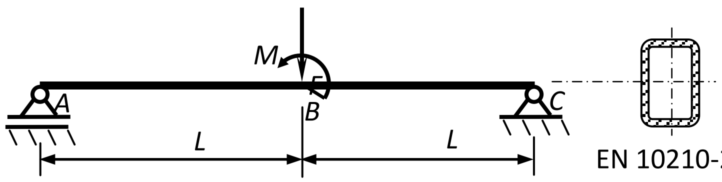
It can be concluded that ***the minimum diameter of the cross-section in order the structure to support the applied loading must be d = 86 mm***.

With this the design has been made.

The next example will regard the same structure with the same loading but the cross-section will be a standard Rectangular hollow section EN 10210-2:2006 (*see Strength of Materials Handbook, page 25, 26 and 27*).

Example 2 – structure with standard cross-section

Design the structure if F = 20 kN, M = 4 kN.m, L = 1 m. The cross-section is a standard Rectangular hollow section EN 10210-2:2006 (*see Strength of Materials Handbook, page 25, 26 and 27*). The material is ductile standard steel S235JR with yielding stress σ_y = 235 MPa. Use a factor of safety FS = 1,2. The internal force V_y is neglected.



EN 10210-2:2006

Steps 1 to 4 for this example are the same as for *Example 1* which means that the critical cross-section is in point B where M_z has maximum – maxM_z = 12 kN.m.

5. *Applying the design formulas*

The design formula is:

$$\frac{|\max M_z|}{S_z} \le \sigma_{all}$$

The allowable stresses σ_{all} is

$$\sigma_{all} = \frac{\sigma_y}{FS} = \frac{235}{1,2} = 195,8\text{MPa}$$

All units in the design formula must be in N, m, Pa which means that

$$\max M_z = 12\text{kN.m} = 12.10^3\,\text{N.m}$$

$$\sigma_{all} = 195,8\text{MPa} = 195,8.10^6\,\text{Pa}$$

Substituting with the values in the design formula follows

$$\frac{|\max M_z|}{S_z} \le \sigma_{all} \;\Rightarrow\; \frac{12.10^3}{S_z} \le 195,8.10^6 \;\Rightarrow\; \frac{12.10^3}{195,8.10^6} \le S_z \;\Rightarrow\; S_z \ge 0,0613.10^{-3}\,\text{m}^3$$

In the standard tables the values of S_z for the different shapes are given in cm^3. So in order to select the proper standard shape S_z has to be converted into cm^3 (*see Strength of Materials Handbook, page 39 if you cannot convert units*):

$$S_z \geq 0{,}0613.10^{-3}\,m^3 \;\Rightarrow\; S_z \geq 0{,}0613.10^{-3}.10^6\,cm^3 \;\Rightarrow\; S_z \geq 0{,}0613.10^3\,cm^3 \;\Rightarrow\; S_z \geq 61{,}3cm^3$$

The shape with the closest S_z is 140×80×4 with S_z = 62,9 cm^3.

It can be concluded that ***the minimum standard Rectangular hollow shape which can be used so that the structure can support the applied loading must be 140×80×4***.

With this the design has been made.